CONDITIONS OF THE ENTRANCE EXAMINATIONS – PHD

(A) **You will receive the text of the exam in pdf form by E-mail at a prearranged time. Please, send an E-mail to** `agostonistvan.elte@gmail.com` **if even after 30 minutes you did not receive the test.**

(B) **During your work:**

- **you MAY use textbooks, course notes or non-programmable calculators;**
- **you SHOULD NOT use external help;**
- **you SHOULD NOT use electronic devices for symbolic computations;**
- **you SHOULD NOT use the internet for search or communication.**

BY WRITING THIS TEST, YOU AGREE TO THE TERMS LISTED ABOVE. Any violation of these terms may result in immediate rejection of your application.

(C) **DURATION: 360 minutes**

(D) **SUBJECTS of the exam**

0. (BMA) BASIC MATHEMATICS
1. (ALG) ALGEBRA
2. (ANA) ANALYSIS (REAL AND COMPLEX)
3. (COM) COMBINATORICS
4. (GEO) GEOMETRY
5. (PRO) PROBABILITY THEORY
6. (DEQ) DIFFERENTIAL EQUATIONS
7. (DGT) DIFFERENTIAL GEOMETRY / TOPOLOGY
8. (FAM) FUNCTIONAL ANALYSIS / MEASURE THEORY
9. (NUM) NUMERICAL ANALYSIS
10. (OPR) OPERATIONS RESEARCH
11. (SET) SET THEORY

(E)

- **You have to do the test on BASIC MATHEMATICS (multiple choice test).**
- **You also have to choose 3 FURTHER TOPICS from subjects 1–11, based on your intended research field (see page 2 of this decription or of the examination paper). Attempt all problems from the chosen topics. Each topic is worth 100 marks.**

(F) **Fill the data page (see page 2 of this description or page 2 of the examination paper) and write the solutions of the multiple choice part of your exam (Basic mathematics) on this sheet. – You may print the data page in advance and fill it during the exam.**

(G) **Send the scanned copy of your solutions (TOGETHER WITH THE DATA PAGE) WITHIN 6 HOURS of receiving the problema sheet to:**

`agostonistvan.elte@gmail.com`

Please, send your solutions in ONE FILE, IF POSSIBLE. A confirmation letter will be sent within the next 24 hours.

(H) **Please, make sure that your presentation is clear and readable. Send us your work even if you have not solved each problem or have only partial results in a certain problem. Unless otherwise stated, you SHOULD JUSTIFY your answers.**

(I) **Besides the page number, please, write on top of each page your NAME, COUNTRY and also indicate the level of the exam (MSc) and the number of the PROBLEM for which you provide a solution on the given page (e.g. Page 5 - PHD/ANALYSIS/2 for the fifth page of your solution, which contains your work on the second problem in analysis).**

DATA PAGE – PHD

Please, fill the table and send it back TOGETHER WITH YOUR SOLUTIONS.
If you do not have a printer, copy the relevant data from the tables by hand.

FULL NAME	
HOME UNIVERSITY	
CITY	
COUNTRY	

Please, choose an area of your future studies (in agreement with your motivation letter) and choose the test subjects accordingly: you have to do the test on the MANDATORY subject, the test of ONE OF THE ELECTIVE subjects and a third test on a subject of YOUR CHOICE. Please, mark your choice by X in the appropriate box. – Note that besides these the test on BASIC MATHEMATICS is also mandatory for every applicant.

AREA OF RESEARCH	BASIC	MANDA-TORY	ELECTIVE	FREE (give subject name)
☐ A. ALGEBRA / NUMBER THEORY	0. BMA	1. ALG	☐ 2. ANA or ☐ 3. COM	
☐ B. ANALYSIS	0. BMA	2. ANA	☐ 6. DEQ or ☐ 8. FAM	
☐ C. APPLIED ANALYSIS	0. BMA	2. ANA	☐ 6. DEQ or ☐ 9. NUM	
☐ D. DISCRETE MATHEMATICS	0. BMA	3. COM	☐ 10. OPR or ☐ 11. SET	
☐ E. GEOMETRY / TOPOLOGY	0. BMA	4. GEO	☐ 7. DGT or ☐ 8. FAM	
☐ F. OPERATIONS RESEARCH	0. BMA	10. OPR	☐ 2. ANA or ☐ 3. COM	
☐ G. PROBABILITY AND STATISTICS	0. BMA	5. PRO	☐ 2. ANA or ☐ 8. FAM	
☐ H. SET THEORY / MATH. LOGIC	0. BMA	11. SET	☐ 1. ALG or ☐ 2. ANA	

SOLUTIONS to the test from BASIC MATHEMATICS

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.

Number of pages sent back (including this data page): ☐

TOPICS OF THE SUBJECTS – PHD

0. (BMA) BASIC MATHEMATICS

Fundamental inference rules based on examples from basic real analysis, arithmetic, combinatorics etc.

1. (ALG) ALGEBRA

Basic notions of elementary algebra: complex numbers, roots of unity, polynomials, matrices, determinants, systems of linear equations. Linear algebra: vector spaces, linear independence, dimension, linear maps, eigenvalue problems, Euclidean spaces, transformations of Euclidean spaces. Abstract algebra: groups (cyclic, symmetric, dihedral, permutation groups, normal subgroups, quotient groups, Sylow subgroups, solvability), rings (rings of polynomials, of matrices, etc, one and two-sided ideals, quotient rings), fields (real, complex, rational, finite, algebraic and transcendental extensions, Galois groups).

2. (ANA) ANALYSIS

Limits of sequences and infinite series. Criteria for convergence. Continuous functions. Derivative, and its applications. Riemann integral and its applications. Primitive function and the Newton–Leibniz formula. Techniques for integration. Sequences and series of functions. Uniform limit. Taylor polynomials and power series. Functions of n variables. Partial derivatives and differentiability. Multiple integrals. Fubini's theorem. Complex differentiability. Complex integration. Residue theorem. Harmonic functions.

3. (COM) COMBINATORICS

Basic counting techiques: permutations, variations, combinations with or without repetition, binomial coefficients and Pascal's triangle, pigeonhole principle, inclusion.exclusion formula, recursions, Fibonacci numbers, Catalan numbers, generating functions. Basic graph theory: directed and undirected graphs, simple graphs, multigraphs, degrees of vertices, connected components, trees and forests, bipartite and multipartite graphs. Eulerian and Hamiltonian paths and circuits. Planar graphs: Euler's formula, estimates on the number of edges, dual graph. Graph colorings: vertex and edge colorings, Brooks' theorem, Vizing's theorem, perfect graphs. Matchings, independent (stable) sets, vertex covers, edge covers, Gallai's theorem, Knig's theorem, Hall's theorem, Tutte's theorem. Extremal graph theory: Turn's theorem, Ramsey theory for finite graphs, Erds-Stone-Simonovits theorem.

4. (GEO) GEOMETRY

Vector operations. Trigonometric functions. Analytic geometry: equations of figures in the plane and space; computing distance and angle between geometrical objects, length, area and volume; algebraic representation of affine transformations and isometries. Convex sets: theorems of Carathéodory, Radon, Helly, Krein–Milman, separation theorems, supporting hyperplanes, convex polytopes and polyhedra, Euler's formula, regular polytopes. Projective space, homogeneos coordinates, cross-ratio, collineations, duality. Quadrics: Euclidean, affine and projective classification, polarity. Spherical and hyperbolic geometry.

5. (PRO) PROBABILITY THEORY

Discrete and absolutely continuous multivariate random variables, expectation, covariance matrix, correlation. Cumulative distribution function, density function of multivariate random variables. Independence of events and random variables. Binomial, negative binomial, hypergeometric, Poisson, normal, exponential, uniform distributions. Conditional probability, law of total probability, Bayes's theorem. Conditional expectation and law of total expectation with respect to an absolutely continuous random variable. Strong and weak laws of large numbers. Central limit theorem. Characteristic function.

6. (DEQ) DIFFERENTIAL EQUATIONS

Separable equations, first-order linear differential equations with initial condition, second-order linear differential equations with initial conditions, stability of two-dimensional linear systems, phase portrait of two-dimensional autonomous systems and stability of the steady states, one-dimensional bifurcations, bifurcation diagram and classification, first-order linear partial differential equations, homogeneous and inhomogeneous heat equations, homogeneous and inhomogeneous wave equations, initial-boundary value problems and their solution with Fourier series.

7. (DGT) DIFFERENTIAL GEOMETRY / TOPOLOGY

Parameterized curves: computation of length, tangents, osculating planes/circles/spheres, evolute, involutes, curvature functions, Frenet frame. Parameterized surfaces: computation of tangent planes, unit normal, fundamental forms, Weingarten map, principal curvatures, Gauss and Minkowski curvature. Smooth manifolds: definition, examples, tangent vectors, the Lie algebra of vector fields, integral curves and the flow of a vector field.

Point set topology: basic notions, separation axioms, countability axioms, connected and path-connected spaces. Fundamental group: definition, homotopy invariance, computation for CW-complexes. Covering maps: classification of covering spaces of a space, universal covering, regular coverings, lifting properties.

8. (FAM) FUNCTIONAL ANALYSIS / MEASURE THEORY

Normed and Banach spaces: elementary properties and examples (finite dimensional spaces, l^p, L^p spaces and spaces of continuous functions), linear operators and functionals, Hahn-Banach theorems, Banach-Steinhaus, closed graph and open mapping theorems, spectral theory of linear operators, compact operators. – Hilbert spaces: elementary properties, orthogonality, Riesz representation theorem, orthogonal and abstract Fourier series, adjoint of an operator and special operator classes (normal, self-adjoint, positive, unitary and isometric operators), Hilbert-Schmidt theory of compact normal operators.

Lebesgue, Lebesgue–Stieltjes and abstract measures and integrals; signed measures and their variations; absolutely continuous and singular measures, Radon–Nykodim derivatives; differentiation of Borel measures; singular and absolutely continuous functions; product measures; L_p spaces, and convolutions.

9. (NUM) NUMERICAL ANALYSIS

Systems of linear and nonlinear equations. Standard factorizations and iterative methods. – Ordinary differential equations. Runge–Kutta methods and linear multistep methods. – Elliptic partial differential eqautions. Standard one and higher dimensional finite difference methods with various boundary conditions. – Time-dependent partial differential eqations. Standard finite difference methods for advection and heat conduction problems.

10. (OPR) OPERATIONS RESEARCH

Modeling using linear and integer programming. LP duality, Farkas Lemma. Basic solutions, the simplex method. Integer solutions of linear programs given by totally unimodular matrices. The assignment problem. Dynamic programming, the knapsack problem, longest and shortest paths. Flows in networks.

11. (SET) SET THEORY

Axioms of set theory. Well ordered sets. Properties. Ordinals, ordinal comparison. Transfinite induction/recursion. Applications. Well ordering theorem. Alephs. Zorn lemma. Cofinality.