Spring 2024

## **CONDITIONS OF THE ENTRANCE EXAMINATIONS – MSC**

- (A) You will receive the text of the exam in pdf form by E-mail at a prearranged time. Please, send an E-mail to agostonistvan.elte@gmail.com if even after 30 minutes you did not receive the test.
- (B) During your work:
  - you MAY use textbooks, course notes or non-programmable calculators;
  - you SHOULD NOT use external help;
  - you SHOULD NOT use electronic devices for symbolic computations;
  - $\circ\,$  you SHOULD NOT use the internet for search or communication.

BY WRITING THIS TEST, YOU AGREE TO THE TERMS LISTED ABOVE. Any violation of these terms may result in immediate rejection of your application.

- (C) DURATION: 360 minutes
- (D) SUBJECTS of the exam
  - 0. (BMA) BASIC MATHEMATICS
  - 1. (ALG) ALGEBRA
  - 2. (ANA) ANALYSIS (REAL AND COMPLEX)
  - 3. (COM) COMBINATORICS
  - 4. (GEO) GEOMETRY
  - 5. (PRO) PROBABILITY THEORY
- (E) You have to do the test on BASIC MATHEMATICS (multiple choice test).
  You also have to choose 3 FURTHER TOPICS from subjects 1–5. Attempt all problems from the chosen topics. Each topic is worth 100 marks.
- (F) Fill the data page (see page 2 of this description or page 2 of the examination paper) and write the solutions of the multiple choice part of your exam (Basic mathematics) on this sheet. – You may print the data page in advance and fill it during the exam.
- (G) Send the scanned copy of your solutions (TOGETHER WITH THE DATA PAGE) WITHIN 6 HOURS of receiving the problem sheet to:

#### agostonistvan.elte@gmail.com

Please, send your solutions in ONE FILE, IF POSSIBLE. A confirmation letter will be sent within the next 24 hours.

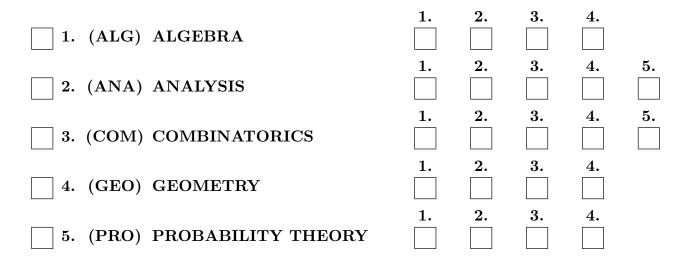
- (H) Please, make sure that your presentation is clear and readable. Send us your work even if you have not solved each problem or have only partial results in a certain problem. Unless otherwise stated, you SHOULD JUSTIFY your answers.
- Besides the page number, please, write on top of each page your NAME, COUNTRY and also indicate the level of the exam (MSc) and the number of the PROBLEM for which you provide a solution on the given page (e.g. Page 5 - MSC/ANALYSIS/2 for the fifth page of your solution, which contains your work on the second problem in analysis).

# DATA PAGE – MSC

Please, fill the table and send it back TOGETHER WITH YOUR SOLUTIONS If you do not have a printer, copy the relevant data from the tables by hand.

FULL NAME	
HOME UNIVERSITY	
CITY	
COUNTRY	

Mark by X problems for which you are sending a solution.



SOLUTIONS of the test from BASIC MATHEMATICS

1	2.	3.	4.	6.		9.	10.

Number of pages sent back (including this data page):

### TOPICS OF THE SUBJECTS – MSC

#### 0. (BMA) BASIC MATHEMATICS

Fundamental inference rules based on examples from basic real analysis, arithmetic, combinatorics etc.

#### 1. (ALG) ALGEBRA

Basic notions of elementary algebra: complex numbers, roots of unity, polynomials, matrices, determinants, systems of linear equations. Linear algebra: vector spaces, linear independence, dimension, linear maps, eigenvalue problems, Euclidean spaces, transformations of Euclidean spaces. Elementary abstract algebra: groups (cyclic, symmetric, dihedral etc.), rings (polynomials, matrices etc.), fields (real, complex, rational, finite).

#### 2. (ANA) ANALYSIS

Limits of sequences and infinite series. Criteria for convergence. Continuous functions. Derivative, and its applications. Riemann integral and its applications. Primitive function and the Newton-Leibniz formula. Techniques for integration. Sequences and series of functions. Uniform limit. Taylor polynomials and power series. Functions of n variables. Partial derivatives and differentiability. Multiple integrals. Fubini's theorem.

#### 3. (COM) COMBINATORICS

Basic counting techiques: permutations, variations, combinations with or without repetition, binomial coefficients and Pascal's triangle, pigeonhole principle, inclusion-exclusion principle, recursions, Fibonacci numbers. Basic graph theory: directed and undirected graphs, simple graphs, multigraphs, degrees of vertices, connected components, trees and forests, bipartite graphs. Eulerian and Hamiltonian paths and circuits. Planar graphs: Euler's formula, estimates on the number of edges, dual graph. Graph colorings: vertex and edge colorings, Brooks' theorem, Vizing's theorem.

#### 4. (GEO) GEOMETRY

Vector operations (linear, affine, convex combinations, dot product, cross product, triple product), their geometrical meaning, identities for them. Trigonometric functions and trigonometric identities. Analytic geometry: equations of figures in the plane and space; computing distance and angle between geometrical objects, length, area and volume; algebraic representation of affine transformations and isometries, computing the image of an object under such a transformation.

#### 5. (PRO) PROBABILITY THEORY

Sampling with and without replacement. Probability by counting. Conditional probability, law of total probability, Bayes's theorem. Discrete and absolutely continuous random variables, expectation, variance. Cumulative distribution function, density function. Binomial, negative binomial, hypergeometric, Poisson, normal, exponential, uniform distributions. Independence of events and random variables. Covariance, correlation. Markov's and Chebyshev's inequality. Weak law of large numbers. Central limit theorem.